

<https://www.linkedin.com/feed/update/urn:li:activity:6528832096750964736>

If  $a^2 + b^2 + c^2 = -2$  and

$$f(x) = \det \begin{pmatrix} 1 + a^2x & (1 + b^2)x & (1 + c^2)x \\ (1 + a^2)x & 1 + b^2x & (1 + c^2)x \\ (1 + a^2)x & (1 + b^2)x & 1 + c^2x \end{pmatrix}, \text{ then } f(x) \text{ is a polynomial}$$

of degree

(a) 2; (b) 3; (c) 0; (d) 1

**Solution by Arkady Alt , San Jose ,California, USA.**

$$\begin{aligned} & \det \begin{pmatrix} 1 + a^2x & (1 + b^2)x & (1 + c^2)x \\ (1 + a^2)x & 1 + b^2x & (1 + c^2)x \\ (1 + a^2)x & (1 + b^2)x & 1 + c^2x \end{pmatrix} = \\ & \det \begin{pmatrix} 1 + a^2x & (1 + b^2)x & 1 + a^2x + (1 + b^2)x + (1 + c^2)x \\ (1 + a^2)x & 1 + b^2x & (1 + a^2)x + 1 + b^2x + (1 + c^2)x \\ (1 + a^2)x & (1 + b^2)x & (1 + a^2)x + (1 + b^2)x + 1 + c^2x \end{pmatrix} = \\ & \det \begin{pmatrix} 1 + a^2x & (1 + b^2)x & x(a^2 + b^2 + c^2 + 2) + 1 \\ (1 + a^2)x & 1 + b^2x & x(a^2 + b^2 + c^2 + 2) + 1 \\ (1 + a^2)x & (1 + b^2)x & x(a^2 + b^2 + c^2 + 2) + 1 \end{pmatrix} = \\ & \det \begin{pmatrix} 1 + a^2x & (1 + b^2)x & 1 \\ (1 + a^2)x - (1 + a^2x) & 1 + b^2x - (1 + b^2)x & 1 - 1 \\ (1 + a^2)x - (1 + a^2x) & (1 + b^2)x - (1 + b^2)x & 1 - 1 \end{pmatrix} = \\ & \det \begin{pmatrix} 1 + a^2x & (1 + b^2)x & 1 \\ x - 1 & -(x - 1) & 0 \\ x - 1 & 0 & 0 \end{pmatrix} = (x - 1)^2 \end{aligned}$$